7.7 Q 31

Solution //

Since the integrand is undefined at $y = -3$ and $y = 3$.

we split the function into two over the limit.

$$\int_0^3 \frac{y\,dy}{\sqrt{9-y^2}} = \lim_{b \to 3^-} \int_0^b \frac{y}{\sqrt{9-y^2}}\,dy$$

$$= \lim_{b \to 3^-} \left(-(9-y^2)^{1/2}\right)\Big|_0^b$$

$$= \lim_{b \to 3^-} -(9-b^2)^{1/2} - (-3)$$

$$= 3.$$

and

$$\int_{-3}^0 \frac{y\,dy}{\sqrt{9-y^2}} = \lim_{b \to -3^+} \int_b^0 \frac{y}{\sqrt{9-y^2}}\,dy$$

$$= \lim_{b \to -3^+} \left(-(9-y^2)^{1/2}\right)\Big|_b^0$$

$$= \lim_{b \to -3^+} (-3) - \left(-(9-b^2)^{1/2}\right)$$

$$= -3$$

as $\left(-(9-b^2)^{1/2}\right) \to 0$ when $b \to \pm 3$

then $$\int_{-3}^3 \frac{y\,dy}{\sqrt{9-y^2}}$$

$$= \int_0^3 \frac{y\,dy}{\sqrt{9-y^2}} + \int_{-3}^0 \frac{y\,dy}{\sqrt{9-y^2}}$$

$$= 3 - 3$$

$$= 0$$

$\therefore$ the integral converges to 0.

Comments,,

In this question, you should be able to integrate the integrand properly.

Also you are required to show good use of the limits when the integrand is split into two.

And you must conclude that the integral converges to a certain value. This should be based on the result you obtained.